error variance at time i. But because y (in contrast to x) is not a sufficient statistic, one cannot expect that the estimate $y_{m/m}$ obtained after $m > 1$ measurements, will be the optimum combination of all these measurements.§ This fact is illustrated in Fig. 1 where the standard deviation in the range estimation error is plotted vs time for the optimal (2 state variable) filter and two stagewise optimal (range only) filters using different measurement intervals. The case considered was the following one

$$\text{initial covariance matrix} = \begin{bmatrix} (200)^2, & 0 \\ 0 \quad , & (0.01)^2 \end{bmatrix}$$

$$\text{initial range} = 20{,}000 \text{ ft}$$

$$\text{velocity} = 100 \text{ ft/sec}$$

$$\text{measurement noise variance} = (10 \text{ ft})^2$$

In addition to the fact that the stagewise optimum filters are clearly not optimum over-all, it will be observed that during some periods of time, better results (range only filters) are obtained with a 10 sec measurement interval than with a 1-sec interval. In other words, during these periods of time, the extra measurements actually result in worse performance!

As another criterion for choosing the gains a_i, we may seek to minimize the sum of the range error variances over the entire measurement schedule (a form of optimum interval estimation). Although this problem is simple to solve when the complete state is being estimated (the solution is the Kalman filter), an analytical solution for the limited state problem appears quite difficult to obtain. This optimal programming problem, of course, can be solved numerically; and the results obtained in this way are compared in Fig. 2 to the stagewise optimum results of Fig. 1 (10 sec measurement interval). It is noted that the performance of these two filters is not appreciably different for the particular problem considered here.

If we now consider the case in which only the terminal uncertainty is of interest, it is in fact possible to find a set of scalar gains a_i that will produce an over-all optimum estimate of the range at the terminal time. The reason for this is that, according to Eqs. (8) and (9), $y_{m/m}$ can be selected as an arbitrarily weighted combination of all the m measurements. The measurement weights, or equivalently, the gains a_i, thus can be chosen to produce the same terminal estimate of y as the optimum 2 state variable filter. Although this scheme could be implemented using the same recursive form as used for the previous schemes, it is important to realize that the intermediate values obtained are simply intermediate values in the estimation of the terminal range and are not useful estimates of the range at intermediate times. In fact, one can often do better by taking $y_{i/i} = y_{i/0} = \rho_i$.

Finally, we wish to point out that better results can be obtained for a fixed number of stored gains by reducing the number of measurements and estimating the complete state. This is illustrated in Fig. 3 where the performance of an optimum 2 state variable filter using 10 measurements, spaced 20 sec apart, is shown to be superior to that of the stagewise optimum range only filter using 20 measurements 10 sec apart (both cases requiring 20 stored gains.)

IV. Conclusions

In conclusion, it can be stated that the selection of a good limited state filter will require a careful statement of the constraints and measure of goodness to be employed. Further work is needed in this important area: to adequately explain some of the results reported here, to obtain possible analytical solutions, and to make a more general study of the penalties in performance that can result from using limited state recursive estimators.

§ The Kalman filter is sometimes "derived" as a superposition of single stage filters. This is valid only when the problem is formulated in the required Markov form.

Inclination of Pressure Orifices in Low-Density Flow

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1. Introduction

An important problem in rarefied gas dynamics is the interpretation given to the pressure measured in a cavity that is vented to the surface through an orifice. It has been found that the cavity pressure can differ markedly from the gas pressure when the temperatures of the cavity and the gas are different, and when the molecular mean free path is comparable to the orifice diameter. This is the so called "Orifice Effect." Previous investigators[1–5] have shown the dependence of this effect on orifice diameter and offer semi-empirical orifice-correction schemes that combine theoretical evaluation of the free molecule limit and correlated experimental data for the transitional flow regime. Recent experimental investigations[6] have shown a relationship between cavity pressure and orifice depth. For the first time the influence of the inclination angle of a pressure tube on the cavity pressure is shown in this Note.

2. Experiments

Experiments were performed with a cooled flat plate model in the hypersonic low-density wind tunnel of the DFVLR Göttingen.[7] The investigation was carried out at Mach numbers of 10 and 20. The freestream Reynolds numbers were in the range 300–1600/cm. The model has a row of nine pressure sampling points parallel to the leading edge. The outside holes, and the center one, were drilled perpendicular to the model surface. The others were inclined at angles between 30° and 80° relative to the leading edge.

3. Results

The static pressure distribution across the model was determined from measurements at the points where the holes are normal to the surface. This provides a reference pressure p_r for each sampling point. The measured pressure p_m normalized with the local reference pressure p_r is plotted in Fig. 1 vs the angle of inclination β. The experimental data show that p_m/p_r is a function of β and Reynolds number Re. The influence of β decreases with increasing Re.

In Fig. 2, p_m/p_r is plotted vs Re for a certain inclination angle β. The extrapolation through the set of the experi-

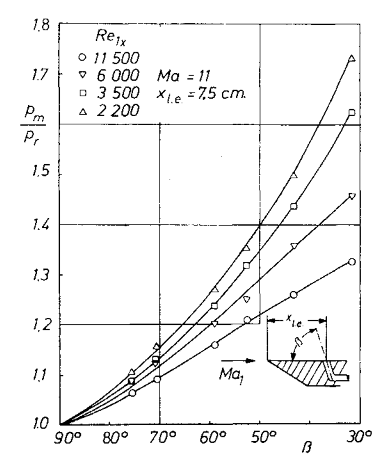

Fig. 1 Measured pressure p_m normalized with the local reference pressure P_r vs the angle of inclination β.

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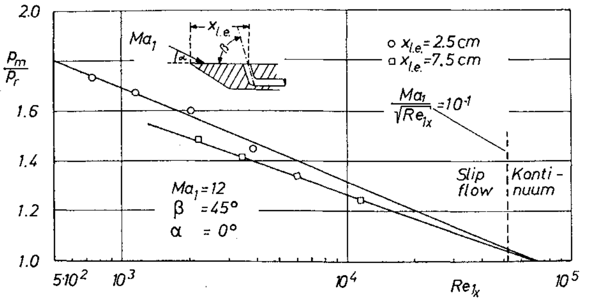

Fig. 2 p_m/p_r **vs** Re **for inclination angle** β.

mental data of two different leading edge distances $x_{l.e.}$ leads to a boundary of the measured effects. This boundary corresponds to the boundary between slip flow and continuum flow defined as $Ma/(\mathrm{Re})^{1/2} = 0.1$.

As the measured effect depends on the rarefaction of the gas, the experimental results are plotted in Fig. 3 over the rarefaction parameter $Kn_w \cdot (x_{l.e.})^{1/2}$ with $Kn_w = \lambda_w/x_{l.e.}$, $\lambda_w = 1.26(R \cdot T_w)^{1/2} \cdot \mu/p_w$ and $x_{l.e.}$ the leading edge distance. All data correlate in this plot for the different test parameters. The curve shows a sudden slope change that corresponds to the change from linear to nearly quadratic characteristic in Fig. 1. At the same point the mean free path in the cavity is of the order of the tube diameter.

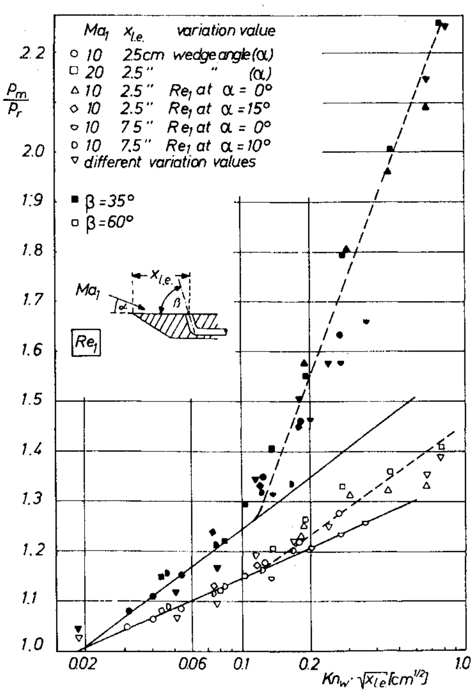

Fig. 3 Experimental results.

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Stability of a Liquid Film

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THE cooling efficiency of a liquid film developed from an ablator or a transpiration cooling system depends on its remaining in contact with the body surface, and its removal in the form of vapor only. If the interface is disturbed from its equilibrium position, waves will form and the dominant liquid layer mass loss mechanism may be the entrainment of liquid droplets into the airstream. In the absence of an external gas, an instability is provided by body forces acting outward from the liquid layer. This situation can be realized under conditions of vehicle deceleration during re-entry where the liquid experiences an effective body force directed away from the vehicle surface. The flow of an external gas has two important effects. The first is the establishment of a velocity profile via the shear at the liquid surface while the second effect is the production of perturbations in the stresses exerted by the gas on the interface due to the appearance of waves that leads to pressure perturbation[1] and shear perturbation instability mechanisms.[2] The purpose of this Note is to study the interaction of tangential body forces with pressure perturbations on the stability of liquid films. The liquid layer is assumed to be thin with one side adjacent to a solid boundary and the other side adjacent to the gas stream. A Cartesian coordinate system is introduced with the $\bar{x}$ axis coinciding with the solid interface and the $\bar{y}$ axis pointing into the liquid. The equilibrium gas-liquid interface is located at $\bar{y} = h$. The body force is directed in such a fashion as to be parallel to the direction of the flight path while the liquid motion is developed from the viscous shear stresses exerted by the gas layer on the liquid layer. The dimensionless velocity of the liquid in the positive x direction is given by

$$U = \gamma y^2 + (1 - \gamma)y \tag{1}$$

where the velocity and the vertical dimension are normalized with the interface velocity U_L, and the liquid depth h. Here

$$\gamma = g \sin\theta h^2/2\nu U_L,$$
$$U_L = (g \sin\theta/2\nu)(2\tau_0 h/\rho g \sin\theta - h^2) \tag{2}$$

with g the body force per unit mass directed in such a way that $g \cos\theta$ is in the positive y-direction and g $\sin\theta$ in the

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